# Numerical simulation of a dotwave system in a harmonic potential well

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*A bath of silicon oil is vibrated and oil droplets are deposited on the surface. Thanks to the vibrations, the droplets do not coalesce, but bounce on the surface.*

*For specific values of amplitude and frequencies, the droplet start to move forward by itself : while bouncing, it create waves with which it interacts.*

*This object has been called a walker, or a dotwave, and exhibits properties reminiscent of an elementary particle, because it is at the same time a particle (the droplet) and a wave (The local standing wave pattern created by the droplet)*

## 1 - Intro

Couder et al. (1) made an experimental setup to study the motion af a dotwave trapped in a harmonic potential well. That is, af if the the walking droplet was attached to a spring which other end is glued at the origin.

Our goal is to simulate that motion

## 2 - Description

The walking droplet moves in a 2D plane. Its position is $\vec{x}(t)$

Following Bush et al. (2), Equation of movement is :

$m\vec{x}'' + D\vec{x}' + k\vec{x} = -mg\nabla h(\vec{x}, t)$ (Eq. 1)

where (Eq. 2) :

$$h(\boldsymbol{x}, t) = \sum_{n=-\infty}^{\lfloor t/T_F \rfloor} \mathrm{AJ}_0(k_F|\boldsymbol{x} - \boldsymbol{x}_p(nT_F)|)\mathrm{e}^{-(t-nT_F)/(T_F M_e)}$$

- The first part of Eq. 1 is a classical harmonic oscillator

- The second part of Eq 1 represents the force that the wave field is applying on the droplet : in the real world, at each bounce, the droplet receives an impulsion proportionnal to the slope of the wave field $h(\vec{x}, t)$. In the simulation, we consider that the field apply this force at each instant on the droplet.

The wave field $h(\vec{x}, t)$ itself is a linear sum of the fields created by the previous bounces of the droplet. (Eq. 2)

Given parameters :

A Amplitude of the Wave Field : depends on the physical parameter of the setup : fluid and gas used (usually silion oil 50 CST and Air) and on the droplet size Rd
k Spring parameter
D Viscous damping

$k_F$ **Wave number of associated wave field**
$T_F$ Faraday period ( = Double of forcing period in the experimental setup)
$M_e$ Memory of the system : when the droplet hits the surface, it creates a localized faraday standing wave pattern that will be non-negligeable for the next Me bounce of the droplet

$J_0$ Bessel Function

## 3 - Calculation

Computed with Matlab, using in-built runge-kutta modified integrated solvers, adapted to Delay Differential Equations.

## 4 - First Result

https://www.youtube.com/watch?v=OoPT_5awqdg

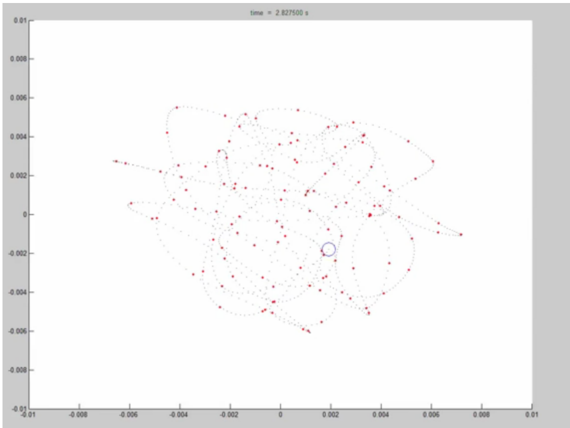
Red Dots every TF (Faraday period, the double of the forcing period) : represent the times when the droplet bounces on the surface.

Blue dots : sampled x10 between red dots along the integration results, ie along the path of the droplet)

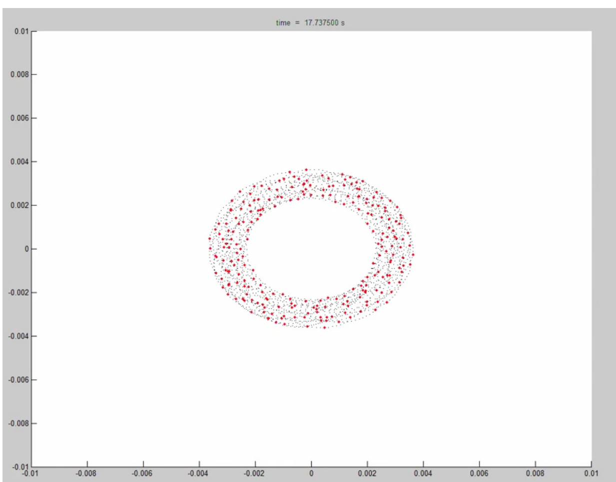
At t = 11 s and t=29s I reset the display to have a better view of what is happening.

Notice 3 periods :

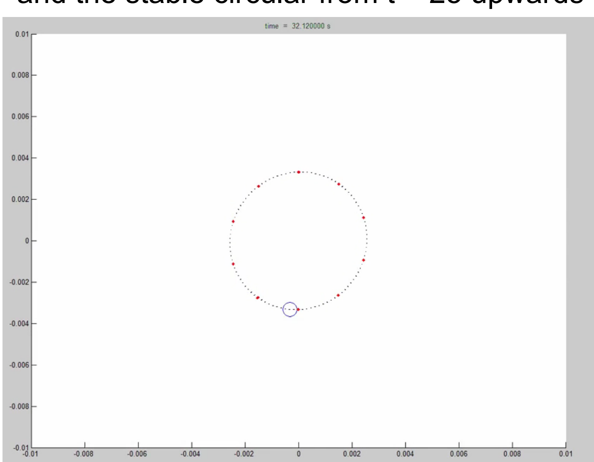
- "chaotic" until t=6 s




- Drifting Elliptic from t= 6 to t = 25




- and the stable circular from t = 25 upwards



Parameters as follows (SI Units) :

```
x_init = 0.001;
y_init = 0.001;
vx_init = 0;
vy_init = 0.005;

% mass of droplet (Computed from density and radius Rd = 0.4 mm)
m = 2.5441e-07;

% Faraday period (For a forcing frequency of 80 Hz)
TF = 0.0250;

% Spring constant

k = 4*pi^2*m/(0.250)^2;

% Viscous Damping
D = 0;

% Wave Force
F = 1.3174e-06;

% Faraday Wave vector module
% Corresponding to a 0.5cm lambda_F wavelenghth
% experimental approximation (Could also be computed from surface wave dispersion relation)
kF = 1250;

% Memory
Me = 150;

% Cutoff  (Waves in Memory)
Cutoff = 16;
```

The cutoff is used when computing the infinite sum on the right part of Eq 2. : we compute only the *Cutoff* last elements of that sum : so the motion of the droplet is influenced by the field created by  the last *Cutoff* bounces only.

(1) Perrard, S., Labousse, M., Miskin, M., Fort, E., & Couder, Y. Effets de quantification d'une association onde-particule soumise à une force centrale.*Résumés des exposés de la 16e Rencontre du Non-Linéaire Paris 2013*, 68
http://dotwave.org/effets-de-quantification-dune-association-onde-particule-soumise-a-une-force-centrale/

(2) Oza, A. U., Rosales, R. R., & Bush, J. W. (2013). A trajectory equation for walking droplets: hydrodynamic pilot-wave theory. *Journal of Fluid Mechanics*,*737*, 552-570.

http://dotwave.org/a-trajectory-equation-for-walking-droplets-hydrodynamic-pilot-wave-theory/